

ficients a_2 , a_1 , and a_0 which minimize the mean square error

$$\sum_{j=1}^n (y_j - p(x_j))^2,$$

over all choices of such coefficients, are obtained by solving the equation

$$[a_2 a_1 a_0] = (X^+ X)^{-1} (X^+ Y),$$

where $X_{ij} = x_i^{3-j}$, for $1 \leq i \leq n$, $1 \leq j \leq 3$, $Y = [y_1 \dots y_n]^T$, and X^+ denotes the Moore-Penrose pseudoinverse of the matrix X . If we let $A = X^+ X$ and $B = X^+ Y$, then we have $A_{ij} = \sum_{k=1}^n x_k^{6-i-j}$, $1 \leq i, j \leq 3$, and the column vector $B_i = \sum_{k=1}^n x_k^{3-i} y_k$, $i=1,2,3$. Once one solves this equation to obtain the optimal value for the leading coefficient, a_2 , the second derivative of the parabola is equal to twice this optimal coefficient. The sign of this second derivative indicates the polarity of the potential spike as well, e.g., a negative second derivative indicates that the peak of the spike is a local maximum for the signal. The above formulae for computing a_2 may be simplified using the symbolic pseudo-inverse of the 3×3 matrix and subsequent matrix multiplication, resulting in the following formulae for a "least squares acceleration filter" which have been implemented to compute this sharpness measure in real time (i.e., the computation of the second derivative at a given point requires only $p(p-1)$ floating point operations):

$$a(p, dt) = dt^4 p(p-1)(2p-1)(3p^2-3p-1)/30,$$

$$b(p, dt) = dt^3 p^2(p-1)^2/4,$$

$$c(p, dt) = dt^2 p(p-1)(2p-1)/6,$$

$$d(p, dt) = dtp(p-1)/2,$$

$$e(p) = p,$$

$$A = ce - d^2,$$

$$B = cd - be,$$

$$C = bd - c^2,$$

$$D = aA + 2bcd - eb^2 - c^3,$$

$$E = [ABC]/D,$$

$$f = [0 dt^2 (2 dt)^2 (3 dt)^2 \dots ((p-1) dt)^2],$$

$$g = [0 dt 2 dt \dots (p-1) dt],$$

$$h = [1 \ 1 \ 1 \dots 1],$$

$$F = 2E[f'g'h^T],$$

and

$$a_2(k - \lfloor (p-1)/2 \rfloor) = F(p)x(k) + F(p-1)x(k-1) + \dots$$

$$F(2)x(k-p+2) + F(1)x(k-p+1),$$

where $\{x_k, k=1, 2, \dots\}$ is the signal being analyzed, p is the number of points used in the parabolic fit (e.g. $p=7$), and dt is the time step of the signal being analyzed. Note that, for a fixed value of p , the filter coefficients F may be computed once and stored for later use in the computation of a_2 from the above FIR filter. The delay in computing a_2 at a given point, which is instilled by using $\lfloor (p-1)/2 \rfloor$ (i.e., the greatest integer less than or equal to $(p-1)/2$) future data points, is only $(p+1)*dt/2$ seconds. For example, with $p=7$ and $dt=1/240$, the delay is $1/60$ sec.

Time-Weighted Averaging

In the methods described above, there are many cases in which it is desirable to compute some background or reference value for a particular signal. By accurately representing the history of the signal, one may improve the method's ability to identify relevant changes which stand out from this background.

In this invention, time-weighted averaging is preferred. A subset of these techniques are able to determine a suitable long time-average of any desired functional of the input signal in a very computationally efficient manner, taking into account the entire recorded history of the signal and using only a minimal amount of computer memory. A particular example of the more general method is that of exponential forgetting, in which the recent signal information is weighted more heavily than the distant past.

The general form referred to as a time-weighted average of a continuous-time signal $\{X_t, t \geq 0\}$ with time-weight $\{f_{t,s}, t \geq 0, 0 \leq s \leq t\}$ is given by $\{m_t, t \geq 0\}$, where

$$M_T = \frac{\int_0^T f_{t,s} X_s ds}{\int_0^T f_{t,s} ds}.$$

The discrete version of this time-average is obtained by simply replacing the integrals in the above definition by the corresponding summations over the index variable s . For certain time-weights, the above formula may be written recursively, in particular, this may be achieved for the case when f is independent of t . If the time-weight $f_{t,s} = e^{-\lambda s}$, then a version of this time-weighted average can be simplified to the exponentially forgetting method that are employed in some of the embodiments of the invention described herein.

Variants on this choice may be useful for certain applications.

For example, by making λ a periodic function of s with a period of one day, the time-weighted average may be used to weight signal information at a particular time of day more heavily than at other times. This may be particularly important for an individual who may commonly have seizure events only during certain times of day.

Also note that the choice of time-weight $f_{t,s} = X_{[t-\delta]}$ gives rise to the usual moving average

$$m_t = \frac{1}{\delta} \int_{t-\delta}^t X_s ds,$$

Where X denotes the indicator function.

In the more general case of "time and state weighted averaging," the weight function, f , may also depend upon the signal X itself. This additional generalization incorporates the case in which the historical average not only depends on the epoch length over which the signal value is averaged, but also upon the signal value over this epoch. This technique may be useful when one desires to allow certain signal characteristics to, by their existence, modify the desired signal background average.

Having thus described the preferred embodiments of the present invention, the following is claimed as new and desired to be secured by Letters Patent:

1. A method of detecting the occurrence of abnormal activity in the brain of a subject, said method comprising the steps of:

- (a) receiving into a signal processor input signals indicative of the subject's brain activity;
- (b) determining ictal components in said input signals by applying to said input signals a first filter configured for extracting and enhancing ictal components from said input signals;